Previous Exam Problems

Problem # 1 (45 points):

Water and gasoline (assumed immiscible) are contained between two infinite concentric cylinders as shown in Figure P1. The diameters of the inner and outer cylinders are $2\ mm$ and $10\ mm$ respectively. The inner cylinder is rotating at an angular speed of $10\ rad/s$ (counter clockwise) and the outer cylinder is fixed. The flow is laminar, incompressible and steady. **Derive an expression for the tangential velocity profile in each layer.** Assume that the flow is axisymmetric and that the velocity and shear stress is continuous across the interface between the two fluids. If needed, take $d_1 = 1\ mm$ and $d_2 = 3\ mm$.

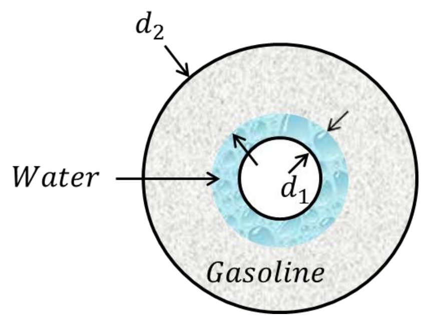

Figure P1

Problem # 2 (55 points):

A swirling vortex can be modeled using the potential flow theory, by superposing a sink and a free vortex at the same point. Consider a swirling vortex (sink strength $m = 12\ m^2/s$ and vortex strength $\Gamma = 5\ m^2/s$ rotating in the counter-clockwise direction) located at (3,4) next to a wall (see Figure P2). **Compute the magnitude and direction of the velocity vector at $(-6, 1)$.** All coordinates are in meters.

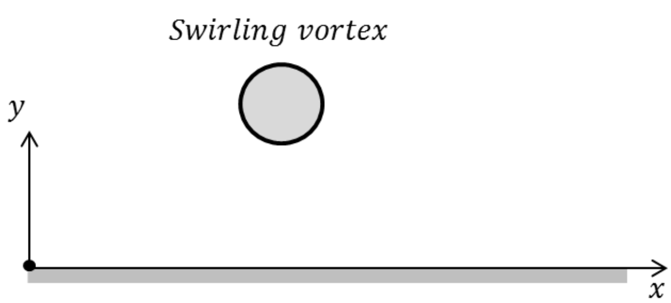

Figure P2

<u>Problem # 3 (30 points):</u>

A Rankine vortex is a realistic model of a real vortex. It consists of a forced vortex of radius r_0 surrounded by a free vortex. The tangential velocity of this vortex can be written as,

$$v_{\theta}= \begin{cases} \frac{\Gamma_0 r}{2\pi r_0^2}, & if\ r \leq r_0 \\ \frac{\Gamma_0}{2\pi r}, & if\ r > r_0 \end{cases}$$

where Γ_0 is a constant and r is the radius from the center of the vortex

a) **What is the circulation $\Gamma(r)$ on a circle of radius r?**
b) **What is the pressure distribution $p(r)$ for $r > r_0$ in an incompressible flow taking $p = p_\infty$ as $r \to \infty$?**

<u>Problem # 4 (40 points):</u>

Water flows between two <u>curved and fixed</u> parallel plates as shown in Figure P4 below. The velocity profile between the two curved plates is assumed to be similar to that of a free vortex as seen in the same figure. The flow is assumed steady and incompressible. Starting from the <u>Navier-Stokes</u> equations in polar coordinates,

a) **Find the pressure difference, Δp, between the upper and lower plate.**
b) **Determine the flow rate, per unit length, between the two plates as a function of Δp**

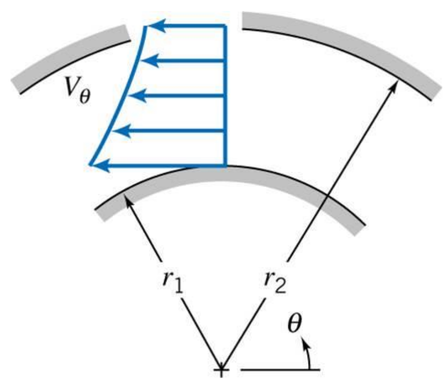

Figure P4

<u>Problem # 5 (50 points):</u>

SAE 30 oil flows between two parallel and inclined plates as shown in Figure P5 below. The lower plate is fixed and the upper plate is being pulled up at a speed $U = 10\ m/s$. The thickness of the upper plate is $d = 1\ mm$. The upper plate is also dragging a layer of water from the other side. The free surface of the water is open to the atmosphere. The flow is assumed steady, incompressible and laminar. Using the Navier-Stokes equations and the data on the figure,

compute the ratio between the flowrate (per unit width) of the oil to that of the water. In which direction are both fluids flowing?

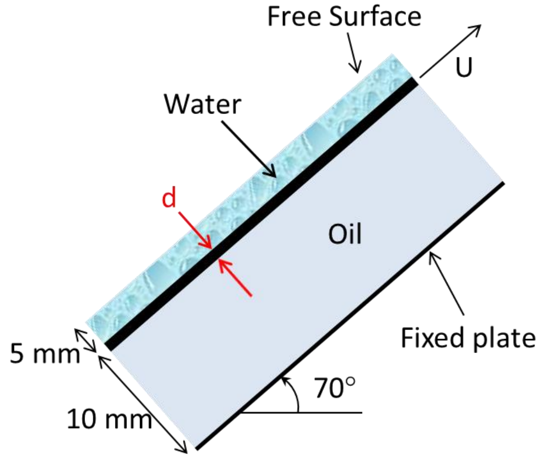

Figure P5